Appearance of Fermion Condensation Quantum Phase Transition in Different Fermi Liquids

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Abstract

We show that the quasiparticle effective mass M^* diverges as a function of the system's density x, $M^* \propto 1/(x - x_{FC})$, when a system approaches the critical point x_{FC} at which the fermion condensation quantum phase transition (FCQPT) occurs. Such behavior is of general form and takes place in both three dimensional systems and two dimensional ones. We demonstrate that a system which has undergone FCQPT and lies close to the critical point x_{FC} can be driven back into the normal Fermi liquid by applying a small magnetic field B. As the field B is reduced, the system is tuned back to the critical point and the effective mass diverges as $M^* \propto 1/\sqrt{B - B_{FC}}$ where B_{FC} is the maximum field at which FCQPT takes place. If the system lies precisely at the critical point then $B_{FC} = 0$. We demonstrate the constancy of the Kadowaki-Woods ratio when approaching the critical point at the fields $B \geq B_{FC}$. Analyzing recent experimental data on the effective mass behavior in a strongly correlated two dimensional fluid ^{3}He, in metallic two dimensional electron systems and in heavy-fermion systems, we show that the observed behavior is in agreement with our consideration. As a result, we may conclude that FCQPT can be conceived of as a universal cause of the strongly correlated regime in different Fermi liquids.

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It is widely believed that unusual properties of the high-temperature superconductors is defined by a quantum phase transition which takes place at temperature $T = 0$ being driven by a control parameter other then temperature, for instance, pressure or the density of mobile charge carriers x. The quantum phase transition occurs at the quantum critical point; in a common case, this point is the end of a line of continuous transitions at $T = 0$. As any phase transition, the quantum phase transition is related to the order parameter which induces a broken symmetry. By now, experimental data show that there are no any broken rotational symmetry or broken translational symmetry, see e.g. [1]. Therefore, the relevant quantum phase transition and its quantum critical point have to possess peculiar features, immediate experimental study of which is of crucial importance for understanding the physics of the high-temperature superconductivity and strongly correlated systems. Unfortunately, it is difficult to study the properties of the high-temperature superconductors, which bear a direct relation to the critical point, because all the corresponding area is occupied by the superconductivity. On the other hand, recent experimental data on different Fermi liquids in the highly correlated regime at the critical point can help to illuminate both the nature of this point and the control parameter by which this point is driven. Experimental facts on strongly correlated high-density two dimensional (2D) ^{3}He [2,3] show that the effective mass diverges when the density at which 2D ^{3}He liquid begins to solidify is approached [3]. Then, sharp increase of the effective mass when the density tends to the critical density of the metal-insulator transition point, which occurs at sufficiently low densities, in a metallic 2D electron system was observed [4]. Note, that there is no ferromagnetic instability in both Fermi systems and the relevant Landau amplitude $F_0^a > -1$ [3,4], in accordance with the almost localized fermion model [5]. At $T \to 0$, another critical point in heavy-fermion metal $YbRh_2Si_2$ is observed [6]. This critical point is driven by magnetic fields B which suppresses the antiferromagnetic order, when B reaches the critical value, $B = B_c$, while the effective mass M^* diverges as $M^* \propto 1/\sqrt{B - B_c}$ [6]. The study of the magnetic field dependence of the coefficients A, γ_0, and χ_0 in the resistivity, $\Delta\rho = A(B)T^2$, specific heat, $C/T = \gamma_0(B)$, and the magnetic ac susceptibility, $\chi_{ac} = \chi_0(B)$, has revealed that $YbRh_2Si_2$ behaves as a true Landau Fermi liquid for $B > B_c$ and the well-known Kadowaki-Woods ratio A/γ_0 [7] is preserved [6]. It is pertinent to note that heavy fermion metals are more likely to be three dimensional (3D) then 2D and nonetheless we observe the same critical point as in the mentioned above strongly correlated Fermi liquids.

In this Report, to study the nature of the discussed critical points, we analyze the appearance of the fermion condensation phase transition (FCQPT) [8] in different 2D and 3D Fermi liquids. We show that at $T \to 0$ FCQPT manifests itself in the divergence of the quasiparticle effective mass M^* as the density x of a system approaches the critical point x_{FC} at which FCQPT takes place, so that $M^* \propto 1/|x - x_{FC}|$. If a Fermi system lies beyond FCQPT point and the density is close to the critical point, then FCQPT can be driven by magnetic fields, while the effective mass diverges as $M^* \propto 1/\sqrt{B - B_{FC}}$ where B_{FC} is the maximum field at which the FCQPT still occurs. As a result, we show that FCQPT can be conceived of as a universal cause of the strongly correlated regime in different Fermi liquids.

We start with a brief consideration of general properties of FCQPT taking, as a case in point, a two-dimensional electron liquid in the superconducting state, when the system has undergone FCQPT [8,9]. At $T = 0$, the ground state energy $E_{gs}[\kappa(\mathbf{p}), n(\mathbf{p})]$ is a functional of the order parameter of the superconducting state $\kappa(\mathbf{p})$ and of the quasiparticle occupation numbers $n(\mathbf{p})$ and is determined by the known equation of the weak-coupling theory of superconductivity (see e.g. [10])

$$E_{gs} = E[n(\mathbf{p})] + \int \lambda_0 V(\mathbf{p}_1, \mathbf{p}_2)\kappa(\mathbf{p}_1)\kappa^*(\mathbf{p}_2)\frac{d\mathbf{p}_1 d\mathbf{p}_2}{(2\pi)^4}. \quad (1)$$

Here $E[n(\mathbf{p})]$ is the ground-state energy of normal Fermi liquid, $n(\mathbf{p}) = v^2(\mathbf{p})$ and $\kappa(\mathbf{p}) = v(\mathbf{p})\sqrt{1-v^2(\mathbf{p})}$. It is assumed that the pairing interaction $\lambda_0 V(\mathbf{p}_1, \mathbf{p}_2)$ is weak. Minimizing E_{gs} with respect to $\kappa(\mathbf{p})$ we obtain the equation connecting the single-particle energy $\varepsilon(\mathbf{p})$ to the superconducting gap $\Delta(\mathbf{p})$

$$\varepsilon(\mathbf{p}) - \mu = \Delta(\mathbf{p})\frac{1-2v^2(\mathbf{p})}{2\kappa(\mathbf{p})}, \tag{2}$$

here μ is the chemical potential. The single-particle energy $\varepsilon(\mathbf{p})$ is determined by the Landau equation [11]

$$\varepsilon(\mathbf{p}) = \frac{\delta E[n(\mathbf{p})]}{\delta n(\mathbf{p})}. \tag{3}$$

The equation for the superconducting gap $\Delta(\mathbf{p})$ takes form

$$\Delta(\mathbf{p}) = -\lambda_0 \int V(\mathbf{p}, \mathbf{p}_1)\kappa(\mathbf{p}_1)\frac{d\mathbf{p}_1}{4\pi^2}. \tag{4}$$

If $\lambda_0 \to 0$, then, the maximum value of the superconducting gap $\Delta_1 \to 0$, and Eq. (2) reduces to that proposed in [12]

$$\varepsilon(\mathbf{p}) - \mu = 0, \text{ if } \kappa(\mathbf{p}) \neq 0, \ (0 < n(\mathbf{p}) < 1); \ \ p_i \leq p \leq p_f \in L_{FC}. \tag{5}$$

At $T = 0$, Eq. (5) defines a new state of Fermi liquid with the fermion condensate (FC) for which the modulus of the order parameter $|\kappa(\mathbf{p})|$ has finite values in L_{FC} range of momenta $p_i \leq p \leq p_f$ occupied by FC, while the superconducting gap can be infinitely small, $\Delta_1 \to 0$ in L_{FC} [8,12,13]. Such a state can be considered as superconducting, with infinitely small value of Δ_1 so that the entropy of this state is equal to zero. This state, created by the quantum phase transition, disappears at $T > 0$. FCQPT can be considered as a "pure" quantum phase transition because it cannot take place at finite temperatures. Therefore, its quantum critical point does not represent the end of a line of continuous transitions at $T = 0$. Equation (5) determines also the critical point of FCQPT, possessing solutions at some density $x = x_{FC}$. Nonetheless, FCQPT has a strong impact on the system's properties up to temperature T_f above which FC effects become insignificant [8,12]. FCQPT does not violate any rotational symmetry or translational symmetry, being characterized by the order parameter $\kappa(\mathbf{p})$. It follows from Eq. (5) that the quasiparticle system brakes into two quasiparticle subsystems: the first one in L_{FC} range is occupied by the quasiparticles with the effective mass $M^*_{FC} \propto 1/\Delta_1$, while the second by quasiparticles with finite mass M^*_L and momenta $p < p_i$. If $\lambda_0 \neq 0$, Δ_1 becomes finite, leading to finite value of the effective mass M^*_{FC} in L_{FC}, which can be obtained from Eq. (2) [8,9]

$$M^*_{FC} \simeq p_F \frac{p_f - p_i}{2\Delta_1}, \tag{6}$$

while the effective mass M^*_L is disturbed weakly. Here p_F is the Fermi momentum. It follows from Eq. (6) that the quasiparticle dispersion can be presented by two straight lines characterized by the effective masses M^*_{FC} and M^*_L respectively. These lines intersect near the binding energy E_0 of electrons which defines an intrinsic energy scale of the system:

$$E_0 = \varepsilon(\mathbf{p}_f) - \varepsilon(\mathbf{p}_i) \simeq \frac{(p_f - p_i)p_F}{M^*_{FC}} \simeq 2\Delta_1. \tag{7}$$

In fact, as it is seen from Eqs. (5) and (6) even at $T = 0$, Fermi liquid with FC presenting highly generated state is absorbed by the superconducting phase transition and never exhibits the dispersionless plateau associated with $M^*_{FC} \to \infty$. As a result, a Fermi liquid beyond the point of FCQPT can be described by two types of quasiparticles characterized by the two finite effective masses M^*_{FC} and M^*_L respectively and by the intrinsic energy scale E_0. It is reasonably safe to suggest that we have come back to the Landau theory by integrating out high energy degrees of freedom and introducing the quasiparticles. The sole difference between the Landau Fermi liquid and Fermi liquid undergone FCQPT is that we have to expand the number of relevant low energy degrees of freedom by adding both a new type of quasiparticles with the effective mass M^*_{FC}, given by Eq. (6), and the energy scale E_0 given by Eq. (7). We have also to bear in mind that the properties of these new quasiparticles of a Fermi liquid with FC cannot be separated from the properties of the superconducting state, as it follows from Eqs. (6) and (7). We may say that the quasiparticle system in the range L_{FC} becomes very "soft" and is to be considered as a strongly correlated liquid. On the other hand, the system's properties and dynamics are dominated by a strong collective effect having its origin in FCQPT and determined by the macroscopic number of quasiparticles in the range L_{FC}. Such a system cannot be disturbed by the scattering of individual quasiparticles and has features of a quantum protectorate [8,14].

Let us assume that FC has just taken place, that is $p_i \to p_f \to p_F$, the deviation $\delta n(p)$ is small, and $\lambda_0 \to 0$. Expanding functional $E[n(p)]$ in Eq. (3) in Taylor's series with respect to $\delta n(p)$ and retaining the leading terms, one obtains taking into account Eq. (5),

$$\mu = \varepsilon(\mathbf{p}, \sigma) = \varepsilon_0(\mathbf{p}, \sigma) + \sum_{\sigma_1} \int F_L(\mathbf{p}, \mathbf{p}_1, \sigma, \sigma_1) \delta n(\mathbf{p_1}, \sigma_1) \frac{d\mathbf{p}_1}{(2\pi)^2}; \;\; p_i \le p \le p_f \in L_{FC}, \tag{8}$$

where $F_L(\mathbf{p}, \mathbf{p}_1, \sigma, \sigma_1) = \delta^2 E / \delta n(\mathbf{p}, \sigma) \delta n(\mathbf{p}_1, \sigma_1)$ is the Landau interaction, and σ denotes the spin states. Both the Landau interaction and the single-particle energy $\varepsilon_0(p)$ are calculated at $n(p) = n_F(p)$. Equation (8) possesses solutions if the Landau amplitude F_L is positive and sufficiently large, so that the integral on the right hand side of Eq. (8) defining the potential energy is large and makes the potential energy prevail over the kinetic energy $\varepsilon_0(\mathbf{p})$ [12]. It is also seen from Eq. (8) that the FC quasiparticles forms a collective state, since their energies are defined by the macroscopical number of quasiparticles within the region $p_i - p_f$, and vice versa. The shape of the spectra is not effected by the Landau interaction, which, generally speaking, depends on the system's properties, including the collective states, impurities, etc. The only thing defined by the interaction is the width of the region $p_i - p_f$, provided the interaction is sufficiently strong to produce the FC phase transition at all. Thus, we can conclude that the spectra related to FC are of universal form. At temperatures $T \ge T_c$, the effective mass M^*_{FC} related to FC is given by [8,9],

$$M^*_{FC} \simeq p_F \frac{p_f - p_i}{4T}. \tag{9}$$

Multiplying both sides of Eq. (9) by $p_f - p_i$ we obtain the energy scale E_0 separating the slow dispersing low energy part, related to the effective mass M^*_{FC}, from the faster dispersing relatively high energy part, defined by the effective mass M^*_L [8,9],

$$E_0 \simeq 4T. \tag{10}$$

It is seen from Eq. (10) that the scale E_0 does not depend on the condensate volume. The single particle excitations are defined according to Eq. (9) by the temperature and by $n(p)$, given by Eq.

(5). Thus, again at $T \geq T_c$, the one-electron spectrum is negligible disturbed by thermal excitations, impurities, etc, and one observes the features of the quantum protectorate [8,14].

FCQPT appears in Fermi liquids, when the effective interaction becomes sufficiently large, as it occurs in ^{3}He at sufficiently high densities and in many-electron systems at relatively low density [15]. Calculations based on simple models show that FC can exist as a stable state separated from the normal Fermi liquid by the phase transition [12]. In ordinary electron liquid this interaction is directly proportional to the dimensionless parameter $r_s \sim 1/p_F a_B$, where a_B is the Bohr radius. If $p_i \to p_F \to p_f$, Eq. (5) determines the point $r = r_{FC}$ at which FCQPT takes place, and $(p_f - p_i)/p_F \sim (r_s - r_{FC})/r_{FC} \sim g_{eff}$, with g_{eff} being the effective coupling constant, which characterizes the interaction in proximity to the point r_{FC} of FCQPT [8,12]. FCQPT precedes the formation of charge-density waves or stripes, which take place at some value $r_s = r_{cdw}$ with $r_{FC} < r_{cdw}$, while the Wigner solidification takes place even at larger values of r_s and leads to insulator [15]. On the other hand, there are charge-density waves, or stripes, in underdoped copper oxides and finally at small doping levels one has insulators [16]. In the same way, the effective mass inevitably diverges as soon as the density x becomes sufficiently large approaching the critical density at which 2D ^{3}He begins to solidify [15], as it was observed in [3].

Recent studies of photoemission spectra discovered an energy scale in the spectrum of low-energy electrons in cuprates, which manifests itself as a kink in the single-particle spectra [17,18]. The spectra in the energy range (-200—0) meV can be described by two straight lines intersecting at the binding energy $E_0 \sim (50-70)$ meV [18]. Then, in underdoped copper oxides, there are the pseudogap phenomenon and the line-shape of single-particle excitations strongly deviates from that of normal Fermi liquid, see, e.g. [19]. While, in the highly overdoped regime slight deviations from the normal Fermi liquid are observed [20]. All these peculiar properties are naturally explained within a model proposed in [8,9,21] and allow to relate the doping level, or the charge carriers density x, regarded as the density of holes (or electrons) per unit area to the density of Fermi liquid with FC. We assume that x_{FC} corresponds to the highly overdoped regime at which FCQPT takes place. In that case, the effective coupling constant $g_{eff} \sim (x - x_{FC})/x_{FC}$. According to the model, the doping level x at $x \leq x_{FC}$ in metals is related to $(p_f - p_i)$ in the following way:

$$g_{eff} \sim \frac{(x_{FC} - x)}{x_{FC}} \sim \frac{p_f - p_i}{p_F} \sim \frac{p_f^2 - p_i^2}{p_F^2}. \tag{11}$$

Now we consider the divergence of the effective mass in 2D and 3D Fermi liquids at $T = 0$, when the density x approaches FCQPT from the side of normal Landau Fermi liquid (LFL). First, we calculate the divergence of M^* as a function of the difference $(x_{FC} - x)$ in case of 2D ^{3}He. For this purpose we use the equation for M^* obtained in [15], where the divergence of the effective mass M^* due to the onset of FC in different Fermi liquids including ^{3}He was predicted

$$\frac{1}{M^*} = \frac{1}{M} + \frac{1}{4\pi^2} \int_{-1}^{1} \int_{0}^{g_0} \frac{v(q(x))}{\left[1 - R(q(x), \omega = 0, g)\chi_0(q(x), \omega = 0)\right]^2} \frac{x dx dg}{\sqrt{1 - x^2}}. \tag{12}$$

Here we adopt the shorthand, $p_F\sqrt{2(1-x)} = q(x)$, with $q(x)$ being the transferred momentum, M is the bare mass, ω is the frequency, $v(q)$ is the bare interaction, and the integral is taken over the coupling constant g from zero to its real value g_0. In Eq. (12), both $\chi_0(q, \omega)$ and $R(q, \omega)$, being the linear response function of noninteracting Fermi liquid and the effective interaction respectively, define the linear response function of the system in question

$$\chi(q,\omega,g)=\frac{\chi_0(q,\omega)}{1-R(q,\omega,g)\chi_0(q,\omega)}. \tag{13}$$

In the vicinity of charge density wave instability, occurring at the density x_{cdw}, the singular part of the function χ^{-1} on the disordered side is of the well-known form, see. e.g. [1]

$$\chi^{-1}(q,\omega,g)\propto(x_{cdw}-x)+(q-q_c)^2+(g_0-g), \tag{14}$$

where $q_c\sim 2p_F$ is the wavenumber of the charge density wave order. Upon substituting Eq. (14) into Eq. (12) and taking the integrals, the equation for the effective mass M^* can be cast into the following form

$$\frac{1}{M^*}=\frac{1}{M}-\frac{C}{\sqrt{x_{cdw}-x}}, \tag{15}$$

with C being some positive constant. It is seen form Eq. (15) that M^* diverges at some point x_{FC}, which is referred to as the critical point at which FCQPT occurs, as a function of the difference $(x_{FC}-x)$

$$M^*\propto\frac{1}{x_{FC}-x}. \tag{16}$$

It follows from the derivation of Eqs. (15) and (16) that the form of these equations is independent of the bare interaction $v(q)$, therefore both of these equations are also applicable to 2D electron liquid or to another Fermi liquid. It is also seen from Eqs. (15) and (16) that FCQPT precedes the formation of charge-density waves. In consequence of this, the effective mass diverges at high densities in case of 2D ^{3}He, and it diverges at low density in case of 2D electron systems, in accordance with experimental facts [3,4]. Note, that in the both cases the difference $(x_{FC}-x)$ has to be positive because x_{FC} represents the solution of Eq. (15). Thus, considering electron systems we have to replace $(x_{FC}-x)$ by $(x-x_{FC})$. In case of 3D system, the effective mass is given by [15]

$$\frac{1}{M^*}=\frac{1}{M}+\frac{p_F}{4\pi^2}\int\limits_{-1}^{1}\int\limits_{0}^{g_0}\frac{v(q(x))xdxdg}{\left[1-R(q(x),\omega=0,g)\chi_0(q(x),\omega=0)\right]^2}. \tag{17}$$

A comparison of Eq. (12) and Eq. (17) shows that there is no fundamental difference between these equations, and along the same way we again arrive at Eqs. (15) and (16). The only difference between 2D electron systems and 3D ones is that FCQPT occurs at densities which are well below those corresponding to 2D systems. While in the bulk ^{3}He, FCQPT cannot probably take place being absorbed by the first order solidification.

Consider the divergence of M^* in external magnetic fields B, when an electron system is located very close to the critical point x_{FC} so that $(x_{FC}-x)/x_{FC}\ll 1$. As it follows from the above, this consideration will be applicable to any 2D or 3D Fermi systems. The application of magnetic field B leads to a weakly polarized state when some levels at the Fermi level are occupied by quasiparticles with the ordered spins. These levels are located in the momentum range δp given by

$$\frac{p_F\delta p}{M^*}\sim B\mu_{eff}, \tag{18}$$

where $\mu_{eff}\sim\mu_B$ is the effective moment. It is seen from Eq. (8) that this polarized state lowers the integral on the right hand side of Eq. (8) by the value δI,

$$\delta I \propto \delta p \propto M^* B. \tag{19}$$

If δI is sufficiently large, Eq. (8) has no solutions and FC vanishes. As a result, we can conclude that $\delta I \propto g_{eff}$. Thus, it is seen form Eq. (19) that the strength of magnetic fields can be used as a control parameter. Let B_{FC} be the magnetic field that suppresses FC so that $p_f \to p_F \to p_I$, then it follows from Eq. (11)

$$(B - B_{FC})M^* \propto g_{eff} \propto \frac{x - x_{FC}}{x_{FC}}. \tag{20}$$

Upon substituting Eq. (20) into Eq. (16) we obtain

$$M^*(B) \propto \frac{1}{\sqrt{B - B_{FC}}}. \tag{21}$$

Equation (21) shows that by applying a magnetic field $B > B_{FC}$, the system can be driven back into LFL with the effective mass $M^*(B)$ dependent on the magnetic field. It was demonstrated that the constancy of the Kadowaki-Woods ratio is obeyed by systems in the strongly correlated regime when the effective mass is sufficiently large [22]. Therefore, we are led to the conclusion that by applying magnetic fields, the system is driven back into LFL, and the constancy of the Kadowaki-Woods ratio is obeyed. Since the resistivity $\Delta\rho \propto (M^*)^2$ [22], we obtain from Eq. (21)

$$\Delta\rho \propto \frac{1}{B - B_{FC}}. \tag{22}$$

Obviously, if the system lies precisely at the critical point, $x = x_{FC}$, then $B_{FC} = 0$. At finite temperatures, there is a temperature $T^*(B)$ at which the polarized state is destroyed. Then, the value of the integral in Eq. (8) is restored, and the system comes back into the state with M^* defined by Eq. (9) and giving rise to the non-Fermi liquid behavior (NFL). For instance, in that case $\Delta\rho \sim T$ [23]. At small fields B, $T^*(B)$ is an increasing function of B because $T^* \sim B\mu_{eff}$. Thus, the higher is the magnetic field B, the higher is the temperature $T^*(B)$, at which the crossover from LFL to NFL occurs.

To explain the nature of the field-induced quantum critical point in $YbRh_2Si_2$, we assume that the electron system of this heavy fermion metal is located very close to the critical point x_{FC}, so that $(x_{FC} - x)/x_{FC} \ll 1$, and see that the obtained above results are in good agreement with experimental facts [6]. We can also safely assume that the electron system of heavy-fermion metals is 3D system. Comparatively small values of T_c observed in these metals rather favor the 3D scenario [24]. A detailed analysis of the discussed experimental facts will be published elsewhere.

To conclude, FCQPT can be conceived of as a universal cause of the strongly correlated regime and of non-Fermi liquid behavior in different Fermi liquids such as 2D ^{3}He liquid of high densities, 2D electron systems of low densities, heavy-fermion metals and the high-temperature superconductors.

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